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Exactly soluble model for self-gravitating D-particles with the wormhole

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Abstract: We consider D-particles coupled to the CGHS dilaton gravity and obtain the exact wormhole geometry and trajectories of D-particles by introducing the exotic matter. The initial static wormhole background is not stable after infalling D-particles due to the classical backreaction of the geometry so that the additional exotic matter source should be introduced for the stability. Then, the traversable wormhole geometry naturally appears and the D-particles can travel through it safely. Finally, we discuss the dynamical evolution of the wormhole throat and the massless limit of D-particles.

Keywords: Classical Theories of Gravity, 2D Gravity.

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1. Introduction

Motivated by a funny fiction, Morris and Thorne [1] have studied the possibility of wormholes and suggested some necessary properties for traversable wormholes. They pointed out that in order to construct a lorentzian wormhole, the exotic matter is required, which is defined as the matter violating the Weak Energy Condition (WEC) [2]. It has been defined by the matter violating the Null Energy Condition (NEC) and it is claimed that its amount is related to the Averaged Null Energy Condition (ANEC) [3].

As a toy model, based on the Callan-Giddings-Harvey-Strominger(CGHS) dilaton gravity model [4], exactly soluble two-dimensional wormholes solutions are obtained [5] by adding ghost fields which are nothing but the exotic matter source. On the other hand, the N-body self-gravitating system in the Jackiw-Teitelboim (JT) model [6] which has the constant curvature scalar has been studied and the closed solutions are obtained for the massive particles [7]. This exactness without approximations may be useful in studying some nonperturbative physics. Recently, self-gravitating N-body motion described by a slightly modified particle action was exactly solved for the asymptotically flat spacetime by using the CGHS model [8].

In this paper, as a natural extension, we would like to study an interesting soluble wormhole model which is defined by D-particles coupled to the CGHS dilaton gravity, where the kinetic term of the usual scalar fields has a wrong sign to incorporate the exotic matter in our starting action. In fact, the D-branes as nonperturbative objects as RR charge carriers [9] have been extensively studied in the string duality [10]. In the present study, however, the D0-branes coupled to the gravity are considered purely in order for the exact solubility of the geometry and the trajectories of particles aiming to see how the particles as a matter behave as time goes on and affect the geometry. The initial static wormhole geometry is deformed by the infalling D-particles so that the additional exotic source should be considered to maintain the wormhole geometry at the latest time, which is in fact due to the classical back reaction of the geometry after infalling D-particles.

In section 2, we shall define our model as D-particles coupled to the dilaton gravity described by the CGHS like model, where the action for the D-particles is written by introducing einbeins. Then, we shall show in section 3 that the wormhole will be unstable after particles pass through it due to the back reaction of the geometry. In order to maintain the wormhole structure, the additional exotic source will be added to the original source in section 4, so the wormhole will be stabilized within some constraints. Finally, summary and discussions are given in section 5.

2. Action and energy-momentum tensors

We start with D-particles coupled to the two-dimensional dilaton gravity [4] with a conformal scalar ghost written by

$$S = S_{DG} + S_g + S_D \,, \tag{2.1a}$$

$$S_{DG} = \frac{1}{2\pi}\int d^2x\sqrt{-g}e^{-2\phi}\left[R+4(\nabla\phi)^2+4\lambda^2\right], \tag{2.1b}$$

$$S_g = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[\frac{1}{2}(\nabla f)^2\right], \tag{2.1c}$$

$$S_D = -\sum_{a=1}^{N}\int d^2x\int d\tau_a\delta^2(x-z_a(\tau_a))m_a e^{-\phi(x)}\sqrt{-g_{\mu\nu}(x)\frac{dz_a^\mu}{d\tau_a}\frac{dz_a^\nu}{d\tau_a}}\,, \tag{2.1d}$$

where $g_{\mu\nu}$ and ϕ are the metric and the dilaton field, and λ^2 is a cosmological constant. The scalar ghost f with the negative kinetic sign is necessary to construct wormholes. Note that e_a, z_a, and m_a are the einbeins, the particle coordinates, and mass for the N-particles, respectively. The Born-Infeld type action [11] for D-particles can be written by an alternative form in terms of the einbein variables,

$$S_D = \frac{1}{2}\sum_{a=1}^{N}\int d^2x\int d\tau_a\delta^2(x-z_a(\tau_a))e_a(\tau_a)\left[e_a^{-2}(\tau_a)g_{\mu\nu}(x)\frac{dz_a^\mu}{d\tau_a}\frac{dz_a^\nu}{d\tau_a}-m_a^2e^{-2\phi(x)}\right], \tag{2.2}$$

for the massless limit in later. Compared with the conventional massive particles, the mass term is effectively interpreted as the dilaton-dependent mass term.

From the action (2.1) and (2.2), the equations of motion for the metric, dilaton, ghost field, einbein, and the coordinates are given by

$$2e^{-2\phi}\left[\nabla_\mu\nabla_\nu\phi-g_{\mu\nu}\left(\Box\phi-(\nabla\phi)^2+\lambda^2\right)\right] = T^g_{\mu\nu}+T^D_{\mu\nu}, \tag{2.3a}$$

$$e^{-2\phi}\left[R-4(\nabla\phi)^2+4\Box\phi+4\lambda^2\right]-\frac{\pi}{\sqrt{-g}}\sum_a\int d\tau_a\delta^2(x-z_a)e_a m_a^2e^{-2\phi} = 0\,, \tag{2.3b}$$

$$\Box f = 0\,, \tag{2.3c}$$

$$e_a^{-2}g_{\mu\nu}(z_a)\frac{dz_a^\mu}{d\tau_a}\frac{dz_a^\nu}{d\tau_a}+m_a^2e^{-2\phi(z_a)} = 0\,, \tag{2.3d}$$

and

$$g_{\mu\nu}(z_a)\frac{d}{d\tau_a}\left(e_a^{-1}\frac{dz_a^\nu}{d\tau_a}\right)+\Gamma_{\mu\alpha\beta}(z_a)e_a^{-1}\frac{dz_a^\alpha}{d\tau_a}\frac{dz_a^\beta}{d\tau_a}-m_a^2e_ae^{-2\phi(z_a)}\frac{\partial\phi}{\partial z_a^\mu} = 0\,, \tag{2.3e}$$

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respectively, where the energy-momentum tensors due to the ghost field and the point masses are written as

$$T^g_{\mu\nu} = -\frac{1}{2}\left(\nabla_\mu f \nabla_\nu f - \frac{1}{2} g_{\mu\nu}(\nabla f)^2\right), \tag{2.4a}$$

$$T^D_{\mu\nu} = \frac{\pi}{\sqrt{-g}} \sum_a \int d\tau_a \delta^2(x - z_a) e_a^{-1} g_{\mu\alpha} g_{\nu\beta} \frac{dz_a^\alpha}{d\tau_a} \frac{dz_a^\beta}{d\tau_a}, \tag{2.4b}$$

and einbein equation of motion (2.3d) was used to eliminate the particle mass term. Then, combining eqs. (2.3a) and (2.3b) yields the following useful relation,

$$\begin{aligned} e^{-2\phi}\left[R + 2\Box\phi\right] &= \frac{\pi}{\sqrt{-g}} \sum_a \int d\tau_a \delta^2(x - z_a) e_a \left[e_a^{-2} g_{\mu\nu} \frac{dz_a^\mu}{d\tau_a} \frac{dz_a^\nu}{d\tau_a} + m_a^2 e^{-2\phi}\right] \\ &= 0\,, \end{aligned} \tag{2.5}$$

which nicely vanishes by using the einbein equations of motion (2.3d) and it is crucial to obtain the exact geometry and particle trajectories without any approximations.

In the conformal gauge defined by $g_{+-} = -(1/2)e^{2\rho}, g_{--} = g_{++} = 0$, where $x^\pm = (x^0 \pm x^1)$, the above equations of motion (2.3) are written as

$$\begin{aligned} 2e^{-2\phi}\left[2\partial_+\phi\partial_-\phi - \partial_+\partial_-\phi + \frac{1}{2}\lambda^2 e^{2\rho}\right] &= \\ &= T^g_{+-} + T^D_{+-}\,, \end{aligned} \tag{2.6}$$

$$\begin{aligned} 8e^{-2(\rho+\phi)}\left[\partial_+\partial_-\rho + 2\partial_+\phi\partial_-\phi - 2\partial_+\partial_-\phi + \frac{1}{2}\lambda^2 e^{2\rho}\right] &- \\ -2\pi e^{-2\rho} \sum_a \int d\tau_a \delta^2(x - z_a) e_a m_a^2 e^{-2\phi} &= 0\,, \end{aligned} \tag{2.7}$$

$$e^{-2\rho}\partial_+\partial_- f = 0\,, \tag{2.8}$$

$$e_a^{-2} e^{2\rho(z_a)} \frac{dz_a^+}{d\tau_a} \frac{dz_a^-}{d\tau_a} - m_a^2 e^{-2\phi(z_a)} = 0\,, \tag{2.9}$$

$$\frac{d}{d\tau_a}\left(e_a^{-1} \frac{dz_a^\pm}{d\tau_a}\right) + 2e_a^{-1} \frac{\partial\rho(z_a)}{\partial z_a^\pm} \frac{dz_a^\pm}{d\tau_a} \frac{dz_a^\pm}{d\tau_a} + 2m_a^2 e_a e^{-2(\rho+\phi)(z_a)} \frac{\partial\phi}{\partial z_a^\mp} = 0\,, \tag{2.10}$$

with the constraint equations,

$$2e^{-2\phi}\left[\partial_\pm\partial_\pm\phi - 2\partial_\pm\rho\partial_\pm\phi\right] = T^g_{\pm\pm} + T^D_{\pm\pm}\,, \tag{2.11}$$

and the energy-momentum tensors (2.4a) are $T^g_{\pm\pm} = -(1/2)\partial_\pm f \partial_\pm f$ and $T^g_{+-} = 0$. The source of D-particles is written as

$$T^D_{\pm\pm} = 2\pi e^{-2\rho} \sum_a \int d\tau_a \delta^2(x - z_a) e_a^{-1} \frac{e^{4\rho}}{4} \frac{dz_a^\mp}{d\tau_a} \frac{dz_a^\mp}{d\tau_a}, \tag{2.12a}$$

$$T^D_{+-} = 2\pi e^{-2\rho} \sum_a \int d\tau_a \delta^2(x - z_a) e_a^{-1} \frac{e^{4\rho}}{4} \frac{dz_a^+}{d\tau_a} \frac{dz_a^-}{d\tau_a}. \tag{2.12b}$$

Note that eq. (2.12b) shows that the particle source is not conformal while it vanishes with the help of eq. (2.9) for the massless case. The key ingredient of the exact solubility is due to eq. (2.5) written as in the conformal gauge $\partial_+\partial_-(\rho - \phi) = 0$, and then the residual symmetry can be fixed by choosing $\rho = \phi$ called Kruskal gauge.

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For the sake of convenience, we now reparametrize as $m_a e_a d\tau_a = d\lambda_a$ for the interesting massive case. The massless limit for D-particles will be discussed in the final section briefly. From eqs. (2.9) and (2.10), after some tedious calculations [8], we get the following first order differential equations,

$$\frac{dz_a^\pm}{d\lambda_a} = A_a^{(\pm)} e^{-2\rho(z_a)}\,, \tag{2.13}$$

where $A_a^{(\pm)}$ are integration constants which satisfy $A_a^{(+)}A_a^{(-)} - 1 = 0$, and we choose $A_a^{(\pm)} > 0$ in order to make $z_a^\pm$ be increasing functions with respect to λ_a. Combining two equations of (2.13), we obtain the trajectories of the particles,

$$z_a^+ = (A_a^{(+)})^2 \left(z_a^- + B_a\right), \tag{2.14}$$

where B_a is an integration constant. The trajectories of the particles are given as straight lines in our coordinates similar to the massless source in the CGHS model.

Using eqs. (2.13) and (2.14), the energy-momentum tensors of the point particles (2.12) in the Kruskal gauge can be obtained as

$$T^D_{++} = \frac{\pi}{2}\sum_a \frac{m_a}{A_a^3}\delta\left(\frac{x^+}{A_a^2} - x^- - B_a\right), \tag{2.15a}$$

$$T^D_{--} = \frac{\pi}{2}\sum_a m_a A_a \delta\left(\frac{x^+}{A_a^2} - x^- - B_a\right), \tag{2.15b}$$

$$T^D_{+-} = \frac{\pi}{2}\sum_a \frac{m_a}{A_a}\delta\left(\frac{x^+}{A_a^2} - x^- - B_a\right), \tag{2.15c}$$

where we renamed $A_a^{(+)}$ to A_a and $A_a^{(-)} = A_a^{-1}$. At first sight, it seems for the energy-momentum tensors to vanish for the massless limit; however, this is not the case since we have already assumed the massive case when we reparametrize the proper time. On the other hand, the solution of the ghost field is $f = f_+(x^+) + f_-(x^-)$ from eq. (2.8).

3. Wormhole and infalling D-particles

In this section, we shall obtain the wormhole geometry with the infalling D-particles by assuming the exotic energy-momentum densities as $T^g_{\pm\pm} = -\lambda^2$ ($f = \sqrt{2}\lambda(x^+ - x^-)$), which is a special choice to get an exact wormhole geometry. Then, integrating eq. (2.6) with the energy-momentum tensor (2.15c), we obtain the metric solution,

$$e^{-2\rho} = a_+(x^+) + a_-(x^-) - \lambda^2 x^+ x^- - \frac{\pi}{2}\sum_a m_a A_a \left(\frac{x^+}{A_a^2} - x^- - B_a\right)\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right), \tag{3.1}$$

where $a_\pm(x^\pm)$ are integration functions determined by the constraints (2.11) as $\partial_\pm\partial_\pm a_\pm(x^\pm) = \lambda^2$. Integrating it, the following metric solution is given by

$$e^{-2\rho} = D + C_+ x^+ + C_- x^- + \frac{1}{2}\lambda^2\left(x^+ - x^-\right)^2 - \frac{\pi}{2}\sum_a m_a A_a \left(\frac{x^+}{A_a^2} - x^- - B_a\right)\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right), \tag{3.2}$$

where C_+, C_-, and D are integration constants.

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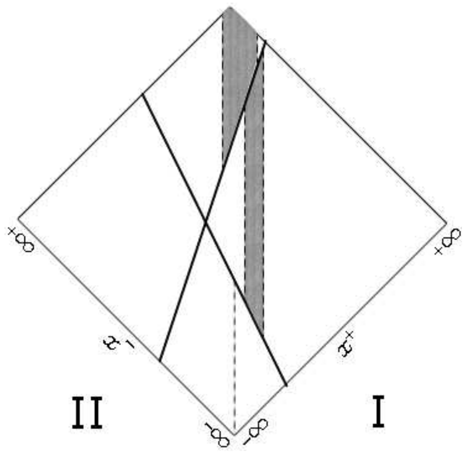

Figure 1: This is a diagram of two-particle case; one starts from our universe (I), the other from the other universe(II). The solid lines denote the particle trajectories, the dotted lines are the horizons, and the shaded regions represent the future trapped regions.

Next, we impose a boundary condition to make a wormhole at the region of $x^{\pm} \to -\infty$ where there are no infalling particles by requiring the horizon coincidence condition as $\partial_+ e^{-2\rho} = \partial_- e^{-2\rho} = 0$. The future and the past horizons are explicitly obtained as

$$0 = C_+ + \lambda^2 \left(x^+ - x^-\right) - \frac{\pi}{2} \sum_a \frac{m_a}{A_a} \theta \left(\frac{x^+}{A_a^2} - x^- - B_a \right), \tag{3.3a}$$

$$0 = C_- - \lambda^2 \left(x^+ - x^-\right) + \frac{\pi}{2} \sum_a m_a A_a \theta \left(\frac{x^+}{A_a^2} - x^- - B_a \right), \tag{3.3b}$$

respectively. So, the static wormhole condition of $x^+ = x^-$ is applied to eq. (3.3) at the region of $x^{\pm} \to -\infty$, then we can fix $C_{\pm}$ and D,

$$C_+ = \frac{\pi}{2} \sum_a^{II} \frac{m_a}{A_a}, \qquad C_- = -\frac{\pi}{2} \sum_a^{II} m_a A_a, \qquad \text{and} \qquad D = \frac{M}{\lambda} - \frac{\pi}{2} \sum_a^{II} m_a A_a B_a, \tag{3.4}$$

where we labeled $\sum_a = \sum_a^I + \sum_a^{II}$, and $\sum_a^I = \sum_{a \in U_I}$ denotes the sum over all the particles starting from our universe while $\sum_a^{II} = \sum_{a \in U_{II}}$ means the sum over all the particles starting from the other universe, i.e., $U_I = \{a | A_a^2 < 1 \text{ or } A_a^2 = 1, B_a > 0\}$ and $U_{II} = \{a | A_a^2 > 1 \text{ or } A_a^2 = 1, B_a < 0\}$. A diagram of two-body case is shown in figure 1; one particle is included in U_I and the other is in U_{II}. As a result, by using the identity, $1 - \theta(x) = \theta(-x)$, the metric solution is represented as

$$\begin{aligned} e^{-2\rho} = \frac{M}{\lambda} + \frac{1}{2}\lambda^2 \left(x^+ - x^-\right)^2 - \frac{\pi}{2} \sum_a^{I} m_a A_a \left(\frac{x^+}{A_a^2} - x^- - B_a \right) \theta \left(\frac{x^+}{A_a^2} - x^- - B_a \right) - \\ - \frac{\pi}{2} \sum_a^{II} m_a A_a \left(x^- - \frac{x^+}{A_a^2} + B_a \right) \theta \left(x^- - \frac{x^+}{A_a^2} + B_a \right), \end{aligned} \tag{3.5}$$

which becomes static wormhole solution [5], $e^{-2\rho} = M/\lambda + (1/2)\left(x^+ - x^-\right)^2$ for $N = 0$.

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Now, we obtain the horizon curves from the metric solution (3.5),

$$0 = \lambda^2 \left(x^+ - x^-\right) - \frac{\pi}{2}\sum_a^I \frac{m_a}{A_a}\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right) + \frac{\pi}{2}\sum_a^{II} \frac{m_a}{A_a}\theta\left(x^- - \frac{x^+}{A_a^2} + B_a\right), \tag{3.6a}$$

$$0 = \lambda^2 \left(x^+ - x^-\right) - \frac{\pi}{2}\sum_a^I m_a A_a\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right) + \frac{\pi}{2}\sum_a^{II} m_a A_a\theta\left(x^- - \frac{x^+}{A_a^2} + B_a\right). \tag{3.6b}$$

The mass (m_a) dependent terms eventually vanish at the asymptotic region of $x^\pm \to -\infty$, and the past and future horizons are coincident because of our boundary condition; however, both horizons are shifted by the infalling particles in our model whereas only the future(past) horizon is shifted for the lightlike infalling from universe I(II). It can be easily seen from the horizon equation (3.6) by simply setting $N = 1$. As an illustration in figure 1, a particle from our universe shifts both horizons to the right where the future horizon is further shifted than the past horizon and the particle from the other universe shifts both horizons to the left, in this case, the past horizon is further shifted than the future horizon. Therefore, the more particles pass through the wormhole, the wider the gap between horizons is.

4. Stability of the wormhole and D-particle trajectories

We have mentioned that the degenerate horizons in the far past are split after particles passed through the wormhole. Thus, it is necessary to improve the exotic energy momentum tensor in order to maintain the wormhole structure even after all particles passed. To do this, the original background exotic(ghost) energy is corrected by $\tilde{T}^g_{\pm\pm} = T^g_{\pm\pm} + \Delta T^g_{\pm\pm}$ such that the added term is $\Delta T^g_{\pm\pm} = \lambda^2\beta_\pm[\theta(x^\pm - x_1^\pm) - \theta(x^\pm - x_0^\pm)]$ and $|\beta_\pm| < 1$ are proportional constants, and $x_0^\pm$ and $x_1^\pm$ satisfying $x_0^\pm < x_1^\pm$ are the coordinates where the additional field is turned on and off. Then, a general metric solution is obtained as

$$\begin{aligned} e^{-2\rho} = {} & D + C_+x^+ + C_-x^- + \frac{1}{2}\lambda^2\left(x^+ - x^-\right)^2 - \frac{\pi}{2}\sum_a m_a A_a\left(\frac{x^+}{A_a^2} - x^- - B_a\right)\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right) + \\ & + \frac{1}{2}\beta_+\lambda^2\left[\left(x^+ - x_0^+\right)^2\theta\left(x^+ - x_0^+\right) - \left(x^+ - x_1^+\right)^2\theta\left(x^+ - x_1^+\right)\right] + \\ & + \frac{1}{2}\beta_-\lambda^2\left[\left(x^- - x_0^-\right)^2\theta\left(x^- - x_0^-\right) - \left(x^- - x_1^-\right)^2\theta\left(x^- - x_1^-\right)\right], \end{aligned} \tag{4.1}$$

where C_+, C_-, and D are integration constants. From the condition of $\partial_+ e^{-2\rho} = \partial_- e^{-2\rho} = 0$, the future and the past horizon are given as

$$\begin{aligned} 0 = {} & C_+ + \lambda^2\left(x^+ - x^-\right) - \frac{\pi}{2}\sum_a \frac{m_a}{A_a}\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right) + \\ & + \beta_+\lambda^2\left[\left(x^+ - x_0^+\right)\theta\left(x^+ - x_0^+\right) - \left(x^+ - x_1^+\right)\theta\left(x^+ - x_1^+\right)\right], \end{aligned} \tag{4.2a}$$

$$\begin{aligned} 0 = {} & C_- - \lambda^2\left(x^+ - x^-\right) + \frac{\pi}{2}\sum_a m_a A_a\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right) + \\ & + \beta_-\lambda^2\left[\left(x^- - x_0^-\right)\theta\left(x^- - x_0^-\right) - \left(x^- - x_1^-\right)\theta\left(x^- - x_1^-\right)\right], \end{aligned} \tag{4.2b}$$

respectively.

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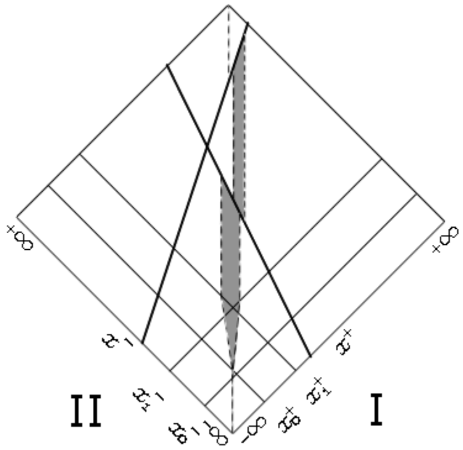

Figure 2: This is a diagram of two-particle case; one starts from our universe (I), the other from the other universe(II), and the additional fields are turned on and off at $x_0^\pm$ and $x_1^\pm$. The thick solid lines denote the particle trajectories, the dotted lines show the horizons, and the shaded regions are the past trapped regions.

Requiring the static wormhole boundary condition that the horizons (4.2) be coincident along with the line $x^+ = x^-$ at $x^\pm \to -\infty$, the same constants $C_\pm$ and D with the previous eq. (3.4) are derived. Next, requiring the same boundary condition at the asymptotic region, $x^\pm \to +\infty$ yields the following relations,

$$\beta_+ = \frac{\pi}{2\lambda^2\left(x_1^+ - x_0^+\right)}\left(\sum_a^{I'}\frac{m_a}{A_a} - \sum_a^{II'}\frac{m_a}{A_a}\right), \tag{4.3a}$$

$$\beta_- = \frac{\pi}{2\lambda^2\left(x_1^- - x_0^-\right)}\left(-\sum_a^{I'} m_a A_a + \sum_a^{II'} m_a A_a\right), \tag{4.3b}$$

where $\sum_a^{I'}$ denotes the sum over all non-static particles starting from our universe and $\sum_a^{II'}$ denotes the sum over all non-static particles starting from the other universe, i.e., $U_{I'} = \{a|A_a^2 < 1\}$ and $U_{II'} = \{a|A_a^2 > 1\}$, where "non-static" means $dz_i^+/dz_i^- = A_i^2 \neq 1$. Note that the static cases represented by $A_a = 1$ and $B_a < 0$ or $B_a > 0$ are canceled out in deriving these relations from eqs. (4.2), which means that only the traveling particles contribute to the shift of the horizon, and consequently, the correction to the exotic energy density is necessary for these non-static infalling cases.

For example, let us consider a single traveling particle($N = 1$) starting from our universe, then eqs. (4.3) read $\beta_+ > 0$ and $\beta_- < 0$, which implies the left-handed(right-handed) correction term ΔT_{++}^g (ΔT_{--}^g) should be negative(positive) to recover the horizon shifts, and the additional densities are explicitly $\Delta T_{++}^g = -(\pi m_1)/(2A_1(x_1^+ - x_0^+))$ and $\Delta T_{--}^g = +(\pi m_1 A_1)/(2(x_1^- - x_0^-))$ in the finite interval $x_0^\pm < x^\pm < x_1^\pm$. The left-handed additional energy-momentum is larger than that of the right-handed one for $x_1^+ = x_1^-$ and $x_0^+ = x_0^-$ since the future horizon is further right-shifted than the past horizon due to the traveling particle starting from our universe as in figure 1. Note that the right-handed source is positive since the past horizon was shifted right.

As a result, the metric solution is given by

$$
\begin{aligned}
e^{-2\rho} = {} & \frac{M}{\lambda} + \frac{1}{2}\lambda^2 \left(x^+ - x^-\right)^2 - \frac{\pi}{2}\sum_a^I m_a A_a \left(\frac{x^+}{A_a^2} - x^- - B_a\right)\theta\left(\frac{x^+}{A_a^2} - x^- - B_a\right) - \\
& - \frac{\pi}{2}\sum_a^{II} m_a A_a \left(x^- - \frac{x^+}{A_a^2} + B_a\right)\theta\left(x^- - \frac{x^+}{A_a^2} + B_a\right) + \\
& + \frac{\pi\left(\sum_a^{I'} m_a - \sum_a^{II'} m_a\right)}{4\lambda^2 A_a \left(x_1^+ - x_0^+\right)}\left[\left(x^+ - x_0^+\right)^2\theta\left(x^+ - x_0^+\right) - \left(x^+ - x_1^+\right)^2\theta\left(x^+ - x_1^+\right)\right] + \\
& + \frac{\pi\left(-\sum_a^{I'} m_a A_a + \sum_a^{II'} m_a A_a\right)}{4\lambda^2\left(x_1^- - x_0^-\right)}\left[\left(x^- - x_0^-\right)^2\theta\left(x^- - x_0^-\right) - \left(x^- - x_1^-\right)^2\theta\left(x^- - x_1^-\right)\right], \qquad (4.4)
\end{aligned}
$$

which becomes again a stable wormhole solution, $e^{-2\rho} = M'/\lambda + (1/2)(x^+ - x^-)^2$ at the region of $x^\pm \to +\infty$, where $M'/\lambda = M/\lambda + (\pi/2)\sum_a^{I'} m_a A_a B_a - (\pi/2)\sum_a^{II'} m_a A_a B_a - (\pi(x_0^+ + x_1^+)/4)(\sum_a^{I'} m_a/A_a - \sum_a^{II'} m_a/A_a) - (\pi(x_0^- + x_1^-)/4)(-\sum_a^{I'} m_a A_a + \sum_a^{II'} m_a A_a)$ while it becomes $e^{-2\rho} = M/\lambda + (1/2)(x^+ - x^-)^2$ at the asymptotic region of $x^\pm \to -\infty$. Considering the one-particle case from our universe ($A_1^2 < 1$), the difference of M between the latest time and the initial time corresponding to the throat change is $\Delta M/\lambda = (\pi m_1/2)[A_1 B_1 - ((x_0 + x_1)/2)A_1^{-1}(1 - A_1^2)]$ for $x_0^+ = x_0^- = x_0$ and $x_1^+ = x_1^- = x_1$. The throat is unchanged, $\Delta M = 0$ especially for $(2A_1^2 B_1)/(1 - A_1^2) = x_0 + x_1$.

We now obtain the i-th particle trajectory in terms of the parameter λ_i by substituting eq. (4.4) into eq. (2.13),

$$
\frac{dz_i^+}{d\lambda_i} = \begin{cases} \sigma_i - \kappa_i z_i^+, & \text{for } \gamma_i^2 = 0 \\ \frac{1}{2}\lambda^2 A_i\left[\gamma_i^{-2}\left(\gamma_i^2 z_i^+ - \zeta_i\right)^2 + \xi_i^2\right], & \text{for } \gamma_i^2 \neq 0 \end{cases}, \qquad (4.5)
$$

where $\gamma_i^2 = [(1 - A_i^2)/A_i^2]^2 + \beta_+\theta(z_i^+ - x_0^+) - \beta_+\theta(z_i^+ - x_1^+) + \beta_- A_i^{-4}\theta(z_i^+/A_i^2 - B_i - x_0^-) - \beta_- A_i^{-4}\theta(z_i^+/A_i^2 - B_i - x_1^-)$, and σ_i, κ_i, ζ_i, and ξ_i^2 are defined by

$$
\begin{aligned}
\sigma_i = {} & \frac{1}{2}\lambda^2 A_i\Bigg[\frac{2M}{\lambda^3} + B_i^2 + \frac{\pi}{\lambda^2}\sum_a^I m_a A_a \left(B_a - B_i\right)\theta\left(z_i^+\left(\frac{1}{A_a^2} - \frac{1}{A_i^2}\right) - \left(B_a - B_i\right)\right) + \\
& + \frac{\pi}{\lambda^2}\sum_a^{II} m_a A_a \left(B_i - B_a\right)\theta\left(z_i^+\left(\frac{1}{A_i^2} - \frac{1}{A_a^2}e\right) - \left(B_i - B_a\right)\right) + \\
& + \beta_+\left(x_0^+\right)^2\theta\left(z_i^+ - x_0^+\right) - \beta_+\left(x_1^+\right)^2\theta\left(z_i^+ - x_1^+\right) + \\
& + \beta_-\left(B_i + x_0^-\right)^2\theta\left(\frac{z_i^+}{A_i^2} - B_i - x_0^-\right) - \beta_-\left(B_i + x_1^-\right)^2\theta\left(\frac{z_i^+}{A_i^2} - B_i - x_1^-\right)\Bigg], \\
\kappa_i = {} & \lambda^2 A_i\Bigg[\frac{\left(1 - A_i^2\right)B_i}{A_i^2} + \frac{\pi}{2\lambda^2}\sum_a^I m_a A_a\left(\frac{1}{A_a^2} - \frac{1}{A_i^2}\right)\theta\left(z_i^+\left(\frac{1}{A_a^2} - \frac{1}{A_i^2}\right) - \left(B_a - B_i\right)\right) + \\
& + \frac{\pi}{2\lambda^2}\sum_a^{II} m_a A_a\left(\frac{1}{A_i^2} - \frac{1}{A_a^2}\right)\theta\left(z_i^+\left(\frac{1}{A_i^2} - \frac{1}{A_a^2}\right) - \left(B_i - B_a\right)\right) + \\
& + \beta_+ x_0^+\theta\left(z_i^+ - x_0^+\right) - \beta_+ x_1^+\theta\left(z_i^+ - x_1^+\right) + \\
& + \beta_-\frac{B_i + x_0^-}{A_i^2}\theta\left(\frac{z_i^+}{A_i^2} - B_i - x_0^-\right) - \beta_-\frac{B_i + x_1^-}{A_i^2}\theta\left(\frac{z_i^+}{A_i^2} - B_i - x_1^-\right)\Bigg],
\end{aligned}
$$

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$\zeta_i = \kappa_i/\lambda^2 A_i$, and $\xi_i^2 = 2\sigma_i/\lambda^2 A_i - \gamma_i^{-2}\zeta_i^2$. Before getting solutions from the above two-differential equations in eq. (4.5), we want to divide the worldline of the i-th particle into some segments intersected by worldlines of other particles. Then, γ_i^2, σ_i, κ_i, ζ_i and ξ_i^2 can be considered as constants in each segment and they are represented by ${\gamma_i^{(r)}}^2$, $\sigma_i^{(r)}$, $\kappa_i^{(r)}$, $\zeta_i^{(r)}$, and ${\xi_i^{(r)}}^2$, where r is an index for the segments. (i) For the case of ${\gamma_i^{(r)}}^2 = 0 = \kappa_i^{(r)}$, the solution is easily obtained as $z_i^+ = \sigma_i^{(r)}(\lambda_i - \Lambda_i^{(r)})$, where $\Lambda_i^{(r)}$ is an integration constant which can be fixed by the continuity with neighbor segments and it is of relevance to choose the origin of the parameter. The solution of z_i^- is easily obtained from eq. (2.14). Note that this solution is adequate only if $\sigma_i^{(r)} > 0$, because we assumed that z_i is an increasing function with respect to λ_i. (ii) For the case of ${\gamma_i^{(r)}}^2 = 0$ and $\kappa_i^{(r)} > 0$, substituting $\sigma_i^{(r)} - \kappa_i^{(r)} z_i^+ = {\eta_i^{(r)}}^{-1}$ into eq. (4.5), we get $dz_i^+/d\lambda_i = {\kappa_i^{(r)}}^{-1}{\eta_i^{(r)}}^{-2} d\eta_i^{(r)}/d\lambda_i = {\eta_i^{(r)}}^{-1}$. Thus, we obtain the particle motion, $z_i^+ = \sigma_i^{(r)}/\kappa_i^{(r)} - {\kappa_i^{(r)}}^{-1}\exp[-\kappa_i^{(r)}(\lambda_i - \Lambda_i^{(r)})]$. Note that this solution is valid for $-\infty < z_i^+ < \sigma_i^{(r)}/\kappa_i^{(r)}$. (iii) For the case of ${\gamma_i^{(r)}}^2 = 0$ and $\kappa_i^{(r)} < 0$, the solution is the same with the case of (ii), but the valid region is different from that of (ii), $\sigma_i^{(r)}/\kappa_i^{(r)} < z_i^+ < +\infty$. (iv) For the case of ${\gamma_i^{(r)}}^2 > 0$ ($\gamma_i^{(r)} > 0$) and ${\xi_i^{(r)}}^2 > 0$ ($\xi_i^{(r)} > 0$), substituting ${\gamma_i^{(r)}}^2 z_i^+ - \zeta_i^{(r)} = \gamma_i^{(r)}\xi_i^{(r)}\tan\eta_i^{(r)}$ into eq. (4.5), we get $z_i^+ = \zeta_i^{(r)}/{\gamma_i^{(r)}}^2 + (\xi_i^{(r)}/\gamma_i^{(r)})\tan\left[(1/2)\lambda^2 A_i \gamma_i^{(r)}\xi_i^{(r)}\left(\lambda_i - \Lambda_i^{(r)}\right)\right]$ with the valid region, $-\infty < z_i^+ < +\infty$. The other cases are similarly obtained, (v) for the case of ${\gamma_i^{(r)}}^2 > 0$ ($\gamma_i^{(r)} > 0$) and ${\xi_i^{(r)}}^2 = 0$ ($\xi_i^{(r)} > 0$), the geodesic equation yields $z_i^+ = \zeta_i^{(r)}/{\gamma_i^{(r)}}^2 - 2/\lambda^2 A_i {\gamma_i^{(r)}}^2\left(\lambda_i - \Lambda_i^{(r)}\right)$. Note that this solution is well-defined everywhere except at the point $z_i^+ = \zeta_i^{(r)}/{\gamma_i^{(r)}}^2$. (vi) For the case of ${\gamma_i^{(r)}}^2 > 0$ ($\gamma_i^{(r)} > 0$) and ${\xi_i^{(r)}}^2 = -{\tilde{\xi}_i^{(r)}}^2 < 0$ ($\tilde{\xi}_i^{(r)} > 0$), the solution is given as $z_i^+ = \zeta_i^{(r)}/{\gamma_i^{(r)}}^2 + (\tilde{\xi}_i^{(r)}/\gamma_i^{(r)})\coth[-(1/2)\lambda^2 A_i\gamma_i^{(r)}\tilde{\xi}_i^{(r)}(\lambda_i - \Lambda_i^{(r)})]$ with the valid ranges, $-\infty < z_i^+ < \zeta_i^{(r)}/{\gamma_i^{(r)}}^2 - \tilde{\xi}_i^{(r)}/\gamma_i^{(r)}$ and $\zeta_i^{(r)}/{\gamma_i^{(r)}}^2 + \tilde{\xi}_i^{(r)}/\gamma_i^{(r)} < z_i^+ < +\infty$. Finally, (vii) for the case of ${\gamma_i^{(r)}}^2 = -{\tilde{\gamma}_i^{(r)}}^2 < 0$ ($\tilde{\gamma}_i^{(r)} > 0$) and ${\xi_i^{(r)}}^2 > 0$ ($\xi_i^{(r)} > 0$), the solution is $z_i^+ = -\zeta_i^{(r)}/{\tilde{\gamma}_i^{(r)}}^2 + (\xi_i^{(r)}/\tilde{\gamma}_i^{(r)})\tanh\left[(1/2)\lambda^2 A_i\tilde{\gamma}_i^{(r)}\xi_i^{(r)}\left(\lambda_i - \Lambda_i^{(r)}\right)\right]$ with the valid range, $-\zeta_i^{(r)}{\tilde{\gamma}_i^{(r)}}^2 - \xi_i^{(r)}/\tilde{\gamma}_i^{(r)} < z_i^+ < -\zeta_i^{(r)}/{\tilde{\gamma}_i^{(r)}}^2 + \xi_i^{(r)}/\tilde{\gamma}_i^{(r)}$. As an illustration, we show the geodesic in figure 3 simply for the two-particle case by assuming the constants as $m_1 = m_2 = M = \lambda = 1$, $A_1^2 = 1/2$, $A_2^2 = 2$, $B_1 = -2$, $B_2 = 1$, $x_0^\pm = -10$, and $x_1^\pm = -5$. Then, the proportional constants are $\beta_\pm = \sqrt{2}\pi/20$, and $\gamma_1^2 = 1 + (\sqrt{2}\pi/20)[\theta(z_1^+ + 10) - \theta(z_1^+ + 5) + 4\theta(z_1^+ + 6) - 4\theta(z_1^+ + 7/2)]$, $\gamma_2^2 = 1/4 + (\sqrt{2}\pi/20)[\theta(z_2^+ + 10) - \theta(z_2^+ + 5) + (1/4)\theta(z_2^+ + 18) - (1/4)\theta(z_2^+ + 8)]$. The intersecting point between the two-particle worldlines is $z^+ = -2$. Figure 3 shows the particle trajectories with respect to the parameter λ. The segments in the particle trajectories are composed of the six segments in each trajectory.

5. Discussion

We studied the soluble D-particle model coupled to the dilaton gravity in 1+1 dimensions in terms of the well-known CGHS model, and especially payed attention to the traversable

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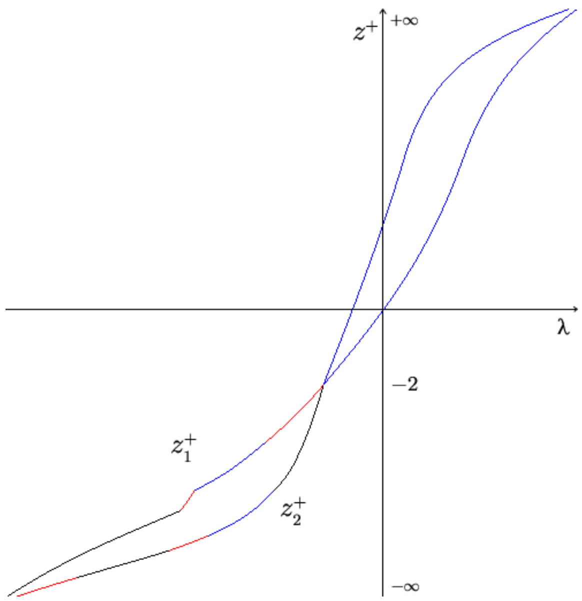

Figure 3: Two particle trajectories which are continuous at each segment.

wormhole construction and its stability. The D-particles can travel from our universe to the other universe with the help of the appropriately corrected exotic matter such that the wormhole geometries appear at both asymptotic regions.

Generically, the coupling term in front of the particle action (2.1d) may be written as $e^{\alpha\phi(x)}$, where α is a constant, then the conventional massive particle action is described for the case of $\alpha = 0$, while the D-particle case does for $\alpha = 1$ as an open string coupling which has been our model. Unfortunately, for the former case, we still do not know how to solve the model exactly; however, the latter case is exactly solved and is still interesting in its own right and we hope it gives some insights to the other physical models.

On the other hand, the wormhole throat is defined by the minimum (radius) of $e^{-2\rho}$ in our model which is regarded as a radial coordinate similar to the higher-dimensional analogue. One can easily find the throat from $0 = \partial_1 e^{-2\rho(x^0,x^1)} = 4\lambda^2 x^1 - (\pi/2)\sum_a^I m_a(A_a + A_a^{-1})\theta((A_a^{-2}-1)x^0 + (A_a^{-2}+1)x^1 - B_a) + (\pi/2)\sum_a^{II} m_a(A_a + A_a^{-1})\theta((1-A_a^{-2})x^0 - (1+A_a^{-2})x^1 + B_a) + \beta_+\lambda^2[(x^0 + x^1 - x_0^+)\theta(x^0 + x^1 - x_0^+) - (x^0 + x^1 - x_1^+)\theta(x^0 + x^1 - x_1^+)] - \beta_-\lambda^2[(x^0 - x^1 - x_0^-)\theta(x^0 - x^1 - x_0^-) - (x^0 - x^1 - x_1^-)\theta(x^0 - x^1 - x_1^-)]$. In a static wormhole state corresponding to $N = 0$ with $\beta_\pm = 0$, the throat radius is constant, which is seen from $e^{-2\rho(x^1=0)} = M/\lambda$. However, in the dynamical unstable case such as simply $N = 1$ and $\beta_\pm = 0$, it is given by $e^{-2\rho(x^1 = x^1_{\text{throat}})} = M/\lambda - (\pi/4)(3x^0 + 25\pi/32\lambda^2)\theta(3x^0 + 25\pi/16\lambda^2)$, where we put $A_1 = 1/2$, $B_1 = 0$ for simplicity, so that the throat radius is suddenly grown up when the particle passes through it, then it will eventually be shrunk to zero and the wormhole disappears. Thus we required $\beta_\pm \neq 0$, in order to stabilize the wormhole if $N \neq 0$.

Although it has not been explicitly shown in this paper, we have considered the massless case with a reparametrization, $e_a d\tau_a = d\lambda_a$. In this case, the particles become light-like and conformal, which is easily seen by substituting $m_a = 0$ into eq. (2.2), and then they are

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independent of the dilaton coupling. And this model can be also exactly solved, which gives similar features. As we pointed out that the massive particles break the residual conformal symmetry in the equation, $\partial_+\partial_-(\rho-\phi)=0$ and the model can not be solved in this frame, however, in our D-particle model the symmetry is naturally maintained by the dilaton coupling. The energy-momentum tensors for the massless limit are interestingly written as a delta-functional type, $T^D_{++}=(\pi/2)\sum_a^{I'} A_a\delta(x^+-z_a^+)$, $T^D_{--}=(\pi/2)\sum_a^{II'} A_a^{-1}\delta(x^--z_a^-)$, and $T^D_{+-}=0$, which appeared in the CGHS model.

The final comment to be mentioned is that the integration constants of eq. (3.4) are determined by the infalling energy-momenta. Note that we just introduced the constant M in the constant D, which seems to be of no relevance to the infalling energy. In fact, the parameter M characterizes the throat of the static wormhole since we have assumed the static wormhole as an initial background geometry, otherwise one should consider the collapsing normal matter which produces the constant M from the Minkowski spacetime. Therefore, it should be related to the infalling energy to make the wormhole geometry. In our analysis, since we introduced the static wormhole as a background, the parameter just describes the throat of the static wormhole before infalling D-particles.

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